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FLUTTER AND VIBRATION OF A HINGELESS HELICOPTER BLADE IN HOVER

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ABSTRACT

In this study, flutter stability and vibration of a uniform bearingless rotor blade in hover with structural coupling is analysed. The blade is modeled as an Euler-Bernoulli beam. The partial differential equations of motion are derived using the Hamilton's principle and solved using the Differential Transform Method, DTM. The computer package Mathematica is used to code the resulting expressions and to calculate the natural frequencies. The effects of the pitch angle and the rotation speed ratio are investigated and the results are compared with the open literature.

INTRODUCTION

A helicopter is an aircraft that uses large diameter rotary wings to provide lift, propulsion, and control. Aerodynamic forces on a helicopter blade are generated by the relative velocity of the rotating wings with respect to air.

Vibratory loads on helicopters arise from a variety of sources such as the main rotor system, the aerodynamic interaction between the rotor and the fuselage, the tail rotor, the engine and the transmission. However, the most significant source of vibration in a helicopter is the main rotor because of the unsteady aerodynamic environment acting on highly flexible rotating blades. The reduction of vibration levels in helicopters below acceptable limits is one of the main problems facing rotorcraft designers. Vibrations lead to passenger discomfort, pilot fatigue, increased noise levels, degradation of weapon effectiveness, and premature failure of aircraft parts.

The field of helicopter aeroelasticity has been a very active area of research during the last 40 years. A comprehensive review paper about this area has been published recently [5].

In this study the blade is modeled as a slender, deformable beam composed of isotropic and homogeneous material. The Euler-Bernoulli hypothesis is assumed to apply and the Blade Element Theory is used to obtain aerodynamic loadS. A semi analytical-numerical technique called the Differential Transform Method, DTM is used to solve the equations. The concept of this method was first introduced by Zhou [9] in 1986 and it was used to solve both linear and nonlinear initial value problems in electric circuit analysis. The method can deal with nonlinear problems so Chiou [8] applied the Taylor transform to solve nonlinear vibration problems. Additionally, the method may be used to solve both ordinary and partial differential equations. Jang et al. [7] applied the two-dimensional differential transform method to the solution of partial differential equations. Hassan [6] adopted the differential transformation method to solve some eigenvalue problems. Since previous studies have shown that the differential transform method is an efficient tool to solve non-linear or parameter varying systems, recently it has gained much attention by several researchers [1-4].

FORMULATION

Aerodynamic Modelling

The elemental lift and drag forces can be written from simple strip theory. The blade is seperated into several stations and the cross-section of the blade at one of the stations and the velocity components, forces and angles at this cross-section are shown in Fig.1.

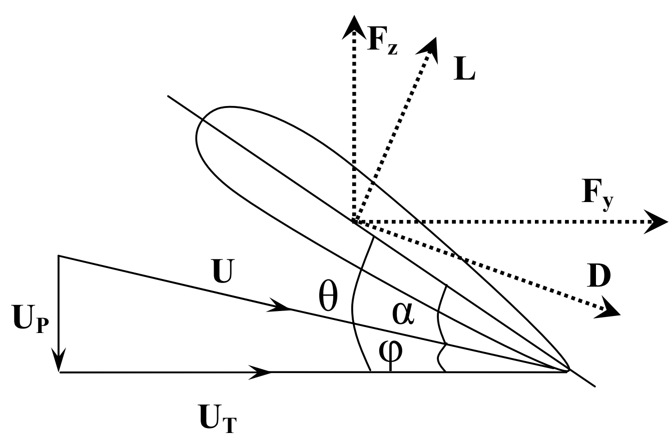

Figure 1. Velocity components, forces and angles at the blade cross section

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Here L is the lift force, D is the drag force, F_z is the vertical force, F_y is the inplane force. U_p, U_T and U are the perpendicular velocity component, tangential velocity component and resultant velocity, respectively. θ, φ and α represent, the geomentric pitch angle, the induced inflow angle and the effective angle of attack, respectively.

Some simplifying assuptions used in the formulation of aerodynamic forces are as follows:

1. Since the tangential velocity component is much greater than the vertical velocity component

$U_T >> U_p$ and $U = \sqrt{U_P^2 + U_T^2} \approx U_T$

2. Since inflow angle is small, $\tan\varphi \cong \varphi = U_P / U_T$. Therefore, the effective angle of attack can be written as
3.

$$\alpha = \theta - \varphi \cong \theta - \frac{U_p}{U_T} \quad (1)$$

The expressions for the sectional lift and the sectional drag forces are given by

$$L = \frac{1}{2}\rho U^2 c C_l = \frac{1}{2}\rho U_T^2 c a \alpha \quad (2)$$

$$D = \frac{1}{2}\rho U^2 c C_d = \frac{1}{2}\rho U_T^2 c a \frac{C_d}{a} \quad (3)$$

Here; ρ is tha air density, a is the lift curve slope, c is the chord length, C_l is the sectional lift coefficient and C_d is the sectional drag coefficient.

Considering the assumptions and Fig.1, the vertical and the inplane forces can be written as follows:

$$dF_z = dLCos\varphi - dDSin\varphi \cong dL - dD\varphi \quad (4)$$

$$dF_y = -(dLSin\varphi + dDCos\varphi) \cong -(dL\varphi + dD) \quad (5)$$

Substituting Eqs.(2) and (3) into Eqs.(4) and (5), we get,

$$dF_z = \frac{\rho a c}{2}\left[\theta U_T^2 - (1 + C_d / a) U_p U_T\right] dx \quad (6)$$

$$dF_y = -\frac{\rho a c}{2}\left[\theta U_P U_T + U_T^2 C_d / a - U_P^2\right] dx \quad (7)$$

Incorporating the airfoil flap and lead displacements (v,w) in the rotating coordinate system, the relative velocities become

$$U_T = \Omega x + \dot{v} \quad (8)$$
$$U_P = \upsilon_i + \dot{w} + \Omega v \beta_{pc} - (\dot{v} + \Omega x)(\theta + \phi) \quad (9)$$

Substituting Eqs. (8) and (9) into the force equations and discarding second order products of displacement velocities such as $\dot{v}^2$, $\dot{w}^2$, $\dot{v}\dot{w}$, the final equations are obtained.

$$F_y = \frac{\rho a c}{2}\left\{ \nu_i^2 - \Omega^2 x^2 \frac{C_D}{a} - \Omega x \nu_i(\theta + \phi) - \left[2\Omega x \frac{C_D}{a} + \nu_i(\theta + \phi)\right]\dot{v} + \left[2\nu_i - \Omega x(\theta + \phi)\right]\dot{w}\right\} \quad (10)$$

$$F_z = \frac{\rho a c}{2}\left\{-\Omega x \nu_i + \Omega^2 x^2(\theta + \phi) - \Omega^2 x \beta_{pc} v + \Omega^2 \frac{xc}{2}(\beta_{pc} + w') - \Omega x \dot{w} + \frac{3}{4} c \Omega x \dot{\phi} + \left[2\Omega x(\theta + \phi) - \nu_i\right]\dot{v} - \frac{c}{4}\ddot{w}\right\} \quad (11)$$

$$M_\phi = -\frac{\rho a c}{2}\left(\frac{c^2}{8}\Omega x \dot{\phi}\right) \quad (12)$$

Here, M_ϕ is the aerodynamic moment, β_{pc} is the precone angle, Ω is the constant rotation speed, ϕ is the torsion angle and v_i is the induced inflow velocity.

Structural Modelling

In this section, Hamilton's principle is used to derive the partial differential equations of motion. In Fig.2, rotor blade cross-section before and after deformation is shown.

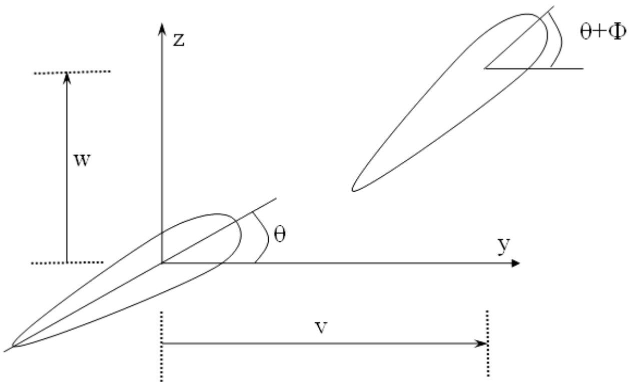

Figure 2. Rotor blade cross-section before and after deformation

By applying Hamilton's principle,

$$\int_{t1}^{t2}(\delta\Im-\delta U+\delta W)dt=0 \tag{13}$$

where $\delta w=\delta v=0$ at t_1 and t_2

differential equations of motion are derived as follows

$\underline{\delta v}$:

$$\begin{aligned}&-(Tv')'+\left[EI_z-(EI_z-EI_y)Sin^2\theta\right]v''''+\\&(EI_z-EI_y)\frac{Sin2\theta}{2}w''''-2m\Omega\beta_{pc}\dot{w}+\\&m(\ddot{v}-\Omega^2 v)=F_y\end{aligned} \tag{14}$$

$\underline{\delta w}$:

$$\begin{aligned}&-(Tw')'+\left[EI_y+(EI_z-EI_y)Sin^2\theta\right]w''''+\\&(EI_z-EI_y)\frac{Sin2\theta}{2}v''''+2m\Omega\beta_{pc}\dot{v}+\\&m\ddot{w}=F_z-m\Omega^2\beta_{pc}x\end{aligned} \tag{15}$$

$\underline{\delta\phi}$:

$$\begin{aligned}&-(GJ\phi')'-k_A^2(T\phi')'+mk_m^2\ddot{\phi}+\\&m\Omega^2(k_{m2}^2-k_{m1}^2)\phi Cos2\theta=M_\phi-\\&m\Omega^2(k_{m2}^2-k_{m1}^2)\frac{Sin2\theta}{2}\end{aligned} \tag{16}$$

Here, k_A is the blade cross section mass radius of gyration, k_{m1} and k_{m2} are the principal mass radii of gyration, m is the mass per unit length and T is the centrifugal force that varies along the spanwise direction and which is expressed as follows

$$T(x)=\int_x^L\rho A\Omega^2(r+x)dx=\rho A\Omega^2\left[r(L-x)+\frac{L^2-x^2}{2}\right] \tag{17}$$

As a byproduct of the Hamiltonian formulation, the associated natural boundary conditions which give the expressions for shear forces and bending moments are also obtained. These boundary conditions are

at $x=0$, $\quad v=w=\phi=0$ (18)

at $x=R$, $\quad \begin{aligned}&v'=w'=0\\&\phi'=0\\&v''=w''=0\\&v'''=w'''=0\end{aligned}$ (19)

STABILITY ANALYSIS

In order to perform the stability analysis, the displacements are written as the summation of steady and perturbation terms,ie.,

$$v(x,t)=v_0(x)+\bar{v}(x,t) \tag{20}$$

$$w(x,t)=w_0(x)+\bar{w}(x,t) \tag{21}$$

$$\phi(x,t)=\phi_0(x)+\bar{\phi}(x,t) \tag{22}$$

Substituting Eqs.(20)-(22) into Eqs.(14)-(16), the perturbation equations are obtained as follows

$$\begin{aligned}&-(T\bar{v}')'+\left[EI_z-(EI_z-EI_y)Sin^2\theta\right]\bar{v}''''+\\&(EI_z-EI_y)\frac{Sin2\theta}{2}\bar{w}''''-2m\Omega\beta_{pc}\dot{\bar{w}}+\\&m(\ddot{\bar{v}}-\Omega^2\bar{v})=\frac{\rho ac}{2}\Big\{-\Omega x v_i\bar{\phi}-\\&\left[2\Omega x\frac{C_D}{a}+v_i\theta\right]\dot{\bar{v}}+\left[2v_i-\Omega x\theta\right]\dot{\bar{w}}\Big\}\end{aligned} \tag{23}$$

$$-\left(T\overline{w}'\right)' + \left[EI_y + \left(EI_z - EI_y\right)Sin^2\theta\right]\overline{w}'''' + \left(EI_z - EI_y\right)\frac{Sin2\theta}{2}\overline{v}'''' + 2m\Omega\beta_{pc}\dot{\overline{v}} + m\ddot{\overline{w}} = \frac{\rho ac}{2}\left\{ \Omega^2 x^2\overline{\phi} - \Omega^2\beta_{pc}x\overline{v} + \frac{xc}{2}\Omega^2\overline{w}' - \Omega x\dot{\overline{w}} + \frac{3}{4}c\Omega x\dot{\overline{\phi}} + \left(2\Omega x\theta - v_i\right)\dot{\overline{v}} - \frac{c}{4}\ddot{\overline{w}}\right\} \tag{24}$$

$$-\left(GJ\overline{\phi}'\right)' - k_A^2\left(T\overline{\phi}'\right)' + mk_m^2\ddot{\overline{\phi}} + m\Omega^2\left(k_{m2}^2 - k_{m1}^2\right)\overline{\phi}Cos2\theta = -\frac{\rho ac}{2}\left(\frac{c^2}{8}\Omega x\dot{\overline{\phi}}\right) \tag{25}$$

For the flutter case, a sinusoidal variation of $v(x,t)$, $w(x,t)$ and $\phi(x,t)$ with a circular natural frequency ω is assumed and the functions are approximated as

$$\overline{v}(x,t) = V(x)e^{i\omega t} \tag{26}$$

$$\overline{w}(x,t) = W(x)e^{i\omega t} \tag{27}$$

$$\overline{\phi}(x,t) = \phi(x)e^{i\omega t} \tag{28}$$

Substituting Eqs.(26)-(28) into Eqs.(23)-(25) results in the following expresions

$$-\frac{d}{dx}\left(T\frac{dV}{dx}\right) + \left[EI_z - \left(EI_z - EI_y\right)Sin^2\theta\right]\frac{d^4V}{dx^4} + \left(EI_z - EI_y\right)\frac{Sin2\theta}{2}\frac{d^4W}{dx^4} - 2i\omega\Omega m\beta_{pc}W - \omega^2 mV - \Omega^2 mV = \frac{\rho ac}{2}\left\{ -\Omega x v_i\phi - i\omega\left[2\Omega x\frac{C_D}{a} + v_i\theta\right]V + i\omega\left[2v_i - \Omega x\theta\right]W\right\} \tag{29}$$

$$-\frac{d}{dx}\left(T\frac{dW}{dx}\right) + \left[EI_y + \left(EI_z - EI_y\right)Sin^2\theta\right]\frac{d^4W}{dx^4} + \left(EI_z - EI_y\right)\frac{Sin2\theta}{2}\frac{d^4V}{dx^4} + 2i\omega\Omega m\beta_{pc}V - \omega^2 mW = \frac{\rho ac}{2}\left\{ \Omega^2x^2\phi - \Omega^2\beta_{pc}xV + \frac{xc}{2}\Omega^2\frac{dW}{dx} - i\omega\Omega xW + i\omega\frac{3}{4}c\Omega x\phi + i\omega\left(2\Omega x\theta - v_i\right)V + \omega^2\frac{c}{4}W\right\} \tag{30}$$

$$-\frac{d}{dx}\left(GJ\frac{d\phi}{dx}\right) - k_A^2\frac{d}{dx}\left(T\frac{d\phi}{dx}\right) - \omega^2 mk_m^2\phi + m\Omega^2\left(k_{m2}^2 - k_{m1}^2\right)\phi Cos2\theta = -i\omega\frac{\rho ac}{2}\left(\frac{c^2}{8}\Omega x\phi\right) \tag{31}$$

Dimensionless Parameters

The dimensionless parameters that are used in order to simplify the equations of motion can be introduced as follows

$$\xi = \frac{x}{R},\ \delta = \frac{r}{R},\ \gamma = \frac{3\rho acR}{m},\ \lambda_i = \frac{v_i}{\Omega R},$$
$$\overline{c} = \frac{c}{R},\ \overline{\omega} = \frac{\omega}{R},\ V(\xi) = \frac{V}{R},\ W(\xi) = \frac{W}{R} \tag{32}$$

The centrifugal force, T, can be expressed in the nondimensional form by using the first two dimensionless parameters

$$T(\xi) = \rho A\Omega^2\left[\frac{1-\xi^2}{2} + \delta(1-\xi)\right] \tag{33}$$

Using these parameters and Eq.(33), the Eqs.(29)-(31) can be given by

$$-\frac{d}{d\xi}\left\{\left[\frac{1-\xi^2}{2} + \delta(1-\xi)\right]\frac{dV}{d\xi}\right\} + A_1\frac{d^4V}{d\xi^4} + A_2\frac{d^4W}{d\xi^4} + A_3W + A_4V + A_5\xi\phi + A_6\xi V + A_7\xi W = 0 \tag{34}$$

$$-\frac{d}{d\xi}\left\{\left[\frac{1-\xi^2}{2} + \delta(1-\xi)\right]\frac{dW}{d\xi}\right\} + B_1\frac{d^4W}{d\xi^4} + B_2\frac{d^4V}{d\xi^4} + B_3V + B_4W + B_5\xi^2\phi + B_6\xi V + B_7\xi W + B_9\xi\frac{dW}{d\xi} + B_9\xi\phi = 0 \tag{35}$$

$$-C_1\frac{d}{d\xi}\left\{\left[\frac{1-\xi^2}{2} + \delta(1-\xi)\right]\frac{d\phi}{d\xi}\right\} + C_2\frac{d^2\phi}{d\xi^2} + C_3\phi + C_4\xi\phi = 0 \tag{36}$$

where the dimensionless coefficients are

$$A_1 = EI_z - \left(EI_z - EI_y\right)Sin^2\theta$$

$$A_2 = \left(EI_z - EI_y\right)\frac{Sin2\theta}{2}$$

$$A_3 = -2i\overline{\omega}\left(\beta_{pc} + \frac{\gamma}{6}\lambda_i\right)$$

$$A_4 = -\left(1+\overline{\omega}^2\right) + i\overline{\omega}\frac{\gamma}{6}\theta\lambda_i$$

$$A_5 = \frac{\gamma}{6}\lambda_i, \quad A_6 = i\overline{\omega}\frac{\gamma}{3}\frac{C_d}{a}, \quad A_7 = i\overline{\omega}\frac{\gamma}{6}\theta$$

$$B_1 = EI_y + \left(EI_z - EI_y\right)Sin^2\theta$$

$$B_2 = A_2 = \left(EI_z - EI_y\right)\frac{Sin2\theta}{2}$$

$$B_3 = i\overline{\omega}\left(2\beta_{pc} + \frac{\gamma}{6}\lambda_i\right)$$

$$B_4 = -\overline{\omega}^2\left(1+\frac{\overline{c}\gamma}{24}\right), B_5 = -\frac{\gamma}{6}$$

$$B_6 = \frac{\gamma}{6}\left(\beta_{pc} - 2i\omega\theta\right)$$

$$B_7 = i\overline{\omega}\frac{\gamma}{6}, \quad B_8 = -\frac{\overline{c}\gamma}{12}, \quad B_9 = -i\overline{\omega}\frac{\overline{c}\gamma}{8}$$

$$C_1 = \left(\frac{k_A}{R}\right)^2, \quad C_2 = -\frac{GJ}{m\Omega^2 R^4}, \quad C_4 = i\overline{\omega}\frac{\overline{c}^2\gamma}{48}$$

$$C_3 = -\overline{\omega}^2\left(\frac{k_m}{R}\right)^2 + \left[\left(\frac{k_{m2}}{R}\right)^2 - \left(\frac{k_{m1}}{R}\right)^2\right]Cos2\theta$$

The dimensionless boundary conditions are

at $\xi = 0$, $$V = W = \phi = 0 \qquad \frac{dV}{d\xi} = \frac{dW}{d\xi} = 0 \tag{37}$$

at $\xi = 1$, $$\frac{d\phi}{d\xi} = 0 \qquad \frac{d^2V}{d\xi^2} = \frac{d^2W}{d\xi^2} = 0 \qquad \frac{d^3V}{d\xi^3} = \frac{d^3W}{d\xi^3} = 0 \tag{38}$$

THE DIFFERENTIAL TRANSFORM METHOD

The differential transform method is a transformation technique based on the Taylor series expansion and it is a useful tool to obtain analytical solutions of the differential equations. In this method, certain transformation rules are applied and the governing differential equations and the boundary conditions of the system are transformed into a set of algebraic equations in terms of the differential transforms of the original functions and the solution of these algebraic equations gives the desired solution of the problem. It is different from high-order Taylor series method because Taylor series method requires symbolic computation of the necessary derivatives of the data functions and is expensive for large orders. The differential transform method is an iterative procedure to obtain analytic Taylor Series solutions of differential equations.

Consider a function $f(x)$ which is analytic in a domain D and let $x = x_0$ represent any point in D. The function $f(x)$ is then represented by a power series whose center is located at x_0. The differential transform of the function $f(x)$ is given by

$$F[k] = \frac{1}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \tag{39}$$

where $f(x)$ is the original function and $F[k]$ is the transformed function. The inverse transformation is defined as

$$f(x) = \sum_{k=0}^{\infty}(x-x_0)^k F[k] \tag{40}$$

Combining Eqs. (42) and (43), we get

$$f(x) = \sum_{k=0}^{\infty}\frac{(x-x_0)^k}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \tag{41}$$

Considering Eq.(44), it is noticed that the concept of differential transform is derived from Taylor series expansion. However, the method does not evaluate the derivatives symbolically.

In actual applications, the function $f(x)$ is expressed by a finite series and Eq. (44) can be written as follows

$$f(x) = \sum_{k=0}^{m}\frac{(x-x_0)^k}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0} \tag{42}$$

which means that

$f(x)=\sum_{k=m+1}^{\infty}\frac{(x-x_0)^k}{k!}\left(\frac{d^k f(x)}{dx^k}\right)_{x=x_0}$ is negligibly small. Here, the value of m depends on the convergence of the natural frequencies.

Theorems that are frequently used in the transformation procedure are introduced in Table 1 and theorems that are used for boundary conditions are introduced in Table 2.

Table 1. Basic theorems of DTM

Original Function	DTM
$f(x)=g(x)\pm h(x)$	$F[k]=G[k]\pm H[k]$
$f(x)=\lambda g(x)$	$F[k]=\lambda G[k]$
$f(x)=g(x)h(x)$	$F[k]=\sum_{l=0}^{k}G[k-l]H[l]$
$f(x)=\frac{d^n g(x)}{dx^n}$	$F[k]=\frac{(k+n)!}{k!}G[k+n]$
$f(x)=x^n$	$F[k]=\delta(k-n)=\begin{cases}0 & if\ \ k\neq n\\ 1 & if\ \ k=n\end{cases}$

Table 2. DTM theorems for boundary conditions

$x=0$	$\frac{df}{dx}(0)=0$	$F(0)=0$
	$\frac{df}{dx}(0)=0$	$F(1)=0$
	$\frac{d^2 f}{dx^2}(0)=0$	$F(2)=0$
	$\frac{d^3 f}{dx^3}(0)=0$	$F(3)=0$
$x=1$	$f(1)=0$	$\sum_{k=0}^{\infty}F(k)=0$
	$\frac{df}{dx}(1)=0$	$\sum_{k=0}^{\infty}kF(k)=0$
	$\frac{d^2 f}{dx^2}(1)=0$	$\sum_{k=0}^{\infty}k(k-1)F(k)=0$
	$\frac{d^3 f}{dx^3}(1)=0$	$\sum_{k=0}^{\infty}(k-1)(k-2)kF(k)=0$

FORMULATION WITH DTM

In the solution step, the Differential Transform Method is applied to Eqs.(34)-(36) by using the theorems introduced in Table 1 and the following expressions are obtained.

$$\begin{aligned}&A_1(k+1)(k+2)(k+3)(k+4)V[k+4]+\\&A_2(k+1)(k+2)(k+3)(k+4)W[k+4]-\\&(k+1)(k+2)\left(\delta+\frac{1}{2}\right)V[k+2]+\\&\delta(k+1)^2V[k+1]+\\&\left[\frac{k(k+1)}{2}+A_4\right]V[k]+\\&A_6V[k-1]+A_3W[k]+\\&A_7W[k-1]+A_5\phi[k-1]=0\end{aligned}\tag{43}$$

$$\begin{aligned}&B_1(k+1)(k+2)(k+3)(k+4)W[k+4]+\\&B_2(k+1)(k+2)(k+3)(k+4)V[k+4]-\\&(k+1)(k+2)\left(\delta+\frac{1}{2}\right)W[k+2]+\\&\delta(k+1)^2W[k+1]+\\&\left[\frac{k(k+1)}{2}+B_4+kB_8\right]W[k]+\\&B_7W[k-1]+B_3V[k]+B_6V[k-1]+\\&B_9\phi[k-1]+B_5\phi[k-2]=0\end{aligned}\tag{44}$$

$$\begin{aligned}&\left[C_2-C_1\left(\delta+\frac{1}{2}\right)\right](k+1)(k+2)\phi[k+2]+\\&C_1(k+1)^2\delta\phi[k+1]+C_4\phi[k-1]+\\&\left[C_3+C_1\frac{k(k+1)}{2}\right]\phi[k]=0\end{aligned}\tag{45}$$

Considering the theorems in Table 2 and Eqs.(37) and (38), the following expressions can be written and these are used in the computer program written to make the related calculations.

$$V[0] = W[0] = V[1] = W[1] = 0$$

$$V[2] = v2\,,\quad V[3] = v3\,,\quad W[2] = w2\,,\quad W[3] = w3$$

$$\phi[0] = 0\,,\quad \phi[1] = \phi 1$$

where $v2$, $v3$, $w2$, $w3$ and $\phi 1$ are arbitrary constants.

RESULTS AND DISCUSSIONS

The computer package Mathematica is used to write a computer program for the expressions obtained using DTM. The results are compared with the ones in Refs.[10]-[11] and a very good agreement with the figures in these references is obtained.

When the aerodynamic terms in the rotor blade equations are eliminated, the resulting perturbation solutions provide the rotating natural frequencies of the rotor blade motion. For untwisted blades at zero pitch angle $(\theta = 0)$, the equations of motion are uncoupled and the resulting natural frequencies are the uncoupled natural frequencies. In the calculation of the rotating natural frequencies, the nonrotating natural frequencies can be used. In Figs.3 (a)-(c), the relations between these frequencies are shown.

In Fig.4(a)-(b), the first two flap and first two lead-lag uncoupled rotating frequencies are given. In Fig.4(a), values for a soft inplane rotor and in Fig.4(b), values for a stiff inplane rotor are introduced. For the soft inplane rotors, the first rotating lead-lag frequency is less than the nominal rotational frequency of the rotor while for the stiff inplane rotors, the lead-lag frequency is greater than the nominal rotational frequency. In this figure, Ω_0 is the nominal angular velocity and $\overline{\omega}_{NR}$ is the natural frequency that is made dimensionless with respect to the nominal angular velocity.

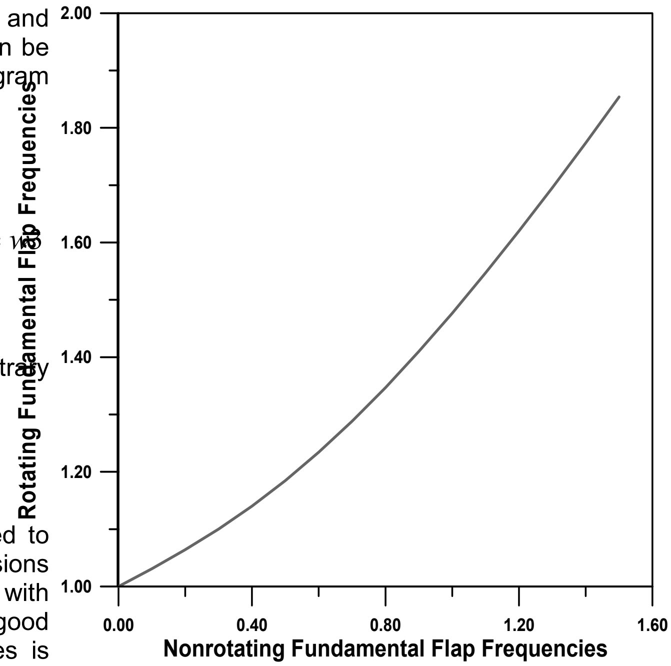

Figure 3(a) Rotating natural flap frequency versus nonrotating flap frequency ($\theta = 0$)

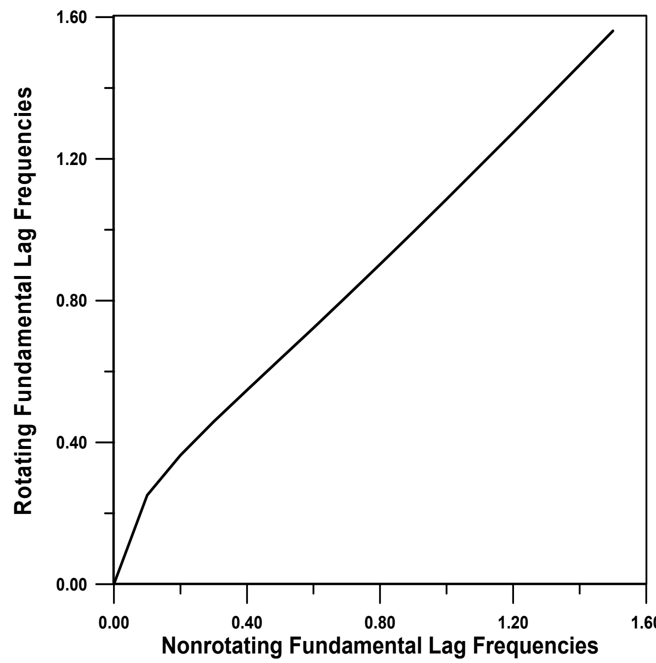

Figure 3(b) Rotating natural lag frequency versus nonrotating lag frequency($\theta = 0$)

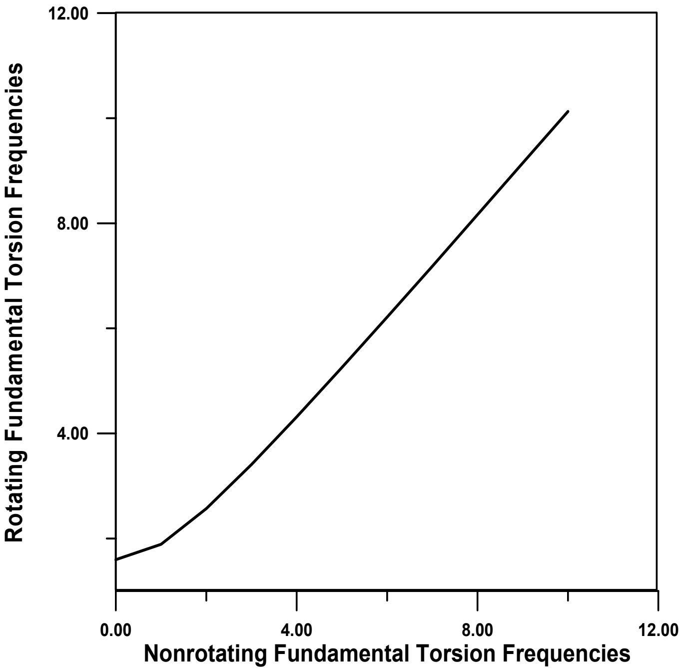

Figure 3(c) Rotating natural torsion frequency versus nonrotating torsion frequency ($\theta = 0$, $\left(k_A / k_m\right)^2 = 1.5$, $k_A / k_m = 0$)

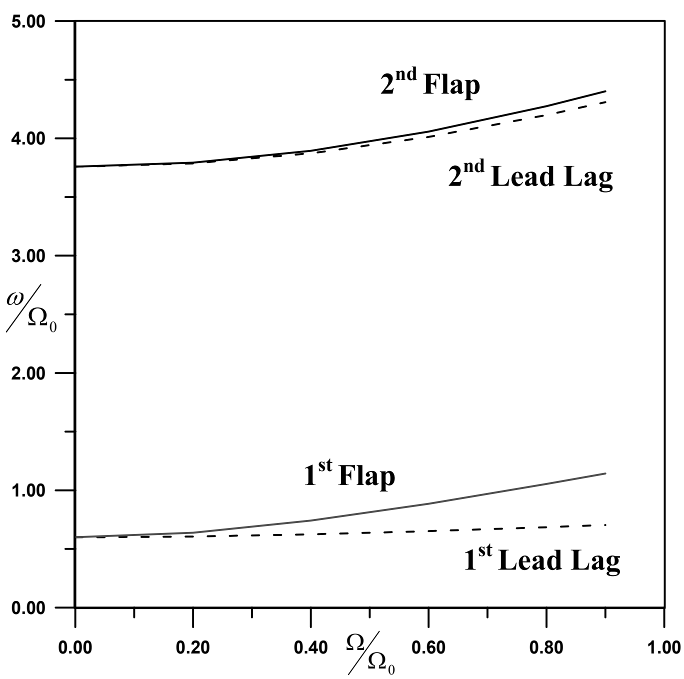

Figure 4(a). Variation of the first two uncoupled natural frequencies with respect to the rotor angular velocity ($\overline{\omega}_{\beta NR} = \overline{\omega}_{\xi NR} = 0.6$).

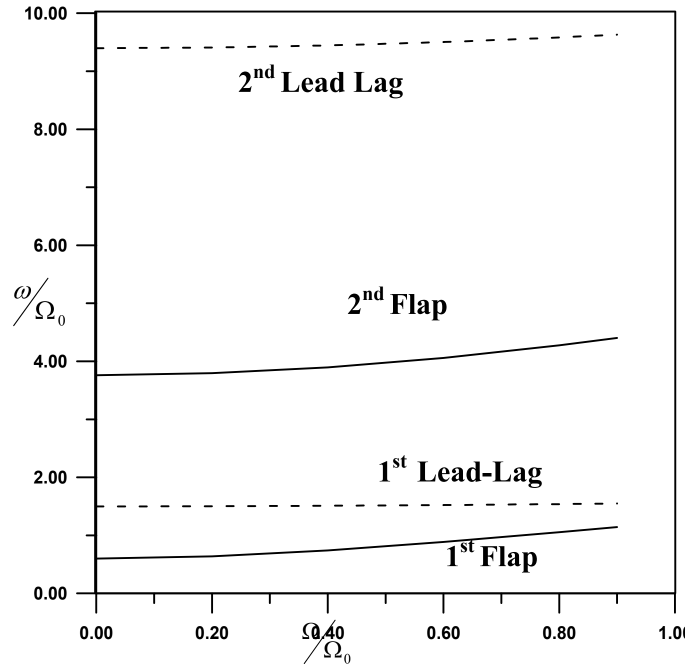

Figure 4(b). Variation of the first two uncoupled natural frequencies with respect to the rotor angular velocity ($\overline{\omega}_{\beta NR} = 0.6$, $\overline{\omega}_{\xi NR} = 1.5$).

In this study, linear formulation is considered. Thus, the torsion motion becomes uncoupled with the flapping and lead-lag motions. However, the flapping and lead-lag motions influence each other. This case is introduced in Fig.5 where the dynamic stability characteristics of the flap and lead-lag bending motions are introduced. The results are given in the root locus form for the first flap and lead-lag modes as the blade pitch angle is increased from zero to 0.5 rad. The figure includes both soft inplane and stiff inplane configurations. As it can be noticed in Fig.5, the damping of the flap mode (real axis component) is high while the damping of the lag mode is low because the lift forces associated with the flapping velocity produce large aerodynamic damping on the flap mode while the drag forces associated with the lead-lag velocity produce small damping on the lead-lag mode. However, the lag mode damping increases with the increasing pitch angle and instability does not occur (none of the real axis components are positive).

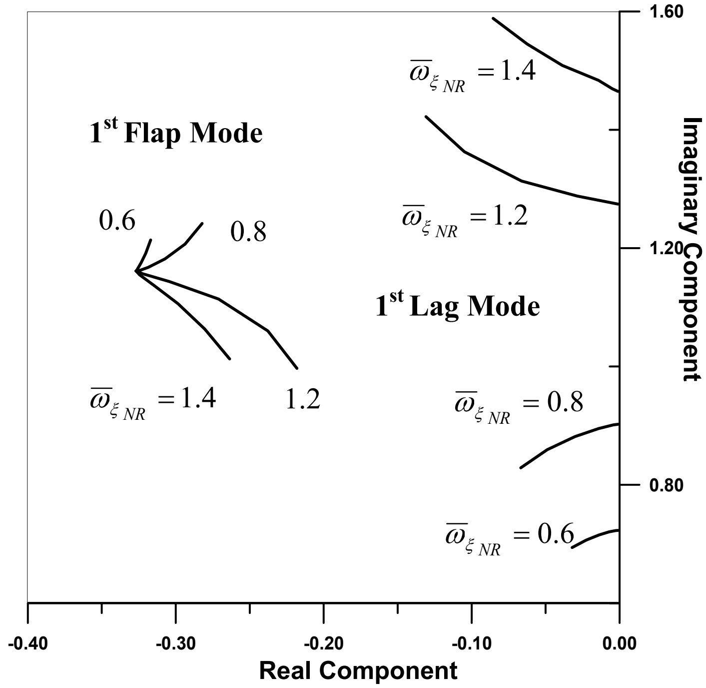

Figure 5. Root locus for the dynamic stability characteristics of the flap and lead-lag bending motions.($\gamma = 5$, $\sigma = 0.05$, $\overline{\omega}_{\beta NR} = 0.6$, $C_d = 0.01$, $a = 2\pi$, $c/R = \pi/40$)

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